Exercise 19

In Exercises 13 to 19, use set theoretic or vector notation or both to describe the points that lie in the given configurations.

The parallelogram whose adjacent sides are the vectors $\mathbf{i} + 3\mathbf{k}$ and $-2\mathbf{j}$

Solution

The vectors of this parallelogram can be written as

$$\begin{aligned} \mathbf{i} + 3\mathbf{k} &= (1, 0, 3) \\ -2\mathbf{j} &= (0, -2, 0). \end{aligned}$$

One is not a constant multiple of the other, so they are linearly independent. A linear combination of these two will span an entire plane.

$$\begin{aligned} \mathbf{r}(s, t) &= s(1, 0, 3) + t(0, -2, 0) \\ &= (s, 0, 3s) + (0, -2t, 0) \\ &= (s, -2t, 3s) \end{aligned}$$

Since we only want the parallelogram, not the whole plane, restrict the values of s and t to $0 \le s \le 1$ and $0 \le t \le 1$, respectively. The set of points in it is described by

$$\{(s, -2t, 3s),\ 0 \le s \le 1,\ 0 \le t \le 1\}.$$